

Combined Deterministic Algorithm and Metaheuristic Technique for Fast and Accurate Resolution of Optimization Problems

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Optimization algorithms are widely developed for solving various engineering problems such as optimal device design, model fitting parameters and the resolution of non-linear system of equations. Whereas deterministic method converges with high accuracy toward the closest optimum of the initial solution, the metaheuristic techniques intends to track the global minimum while avoiding local optima. In order to benefit from the accuracy of the former and the robustness of the latter, we propose to combine the stochastic algorithm and the deterministic process with a strategy based on the behavior of preys and predators. Both techniques runs independently in parallel until the deterministic algorithms converge. After their convergence, the deterministic algorithms are repositioned at the best solutions of the stochastic process whereas the individuals of the metaheuristic method are placed according to a Pareto front between the objective function and their distance to the positions of the deterministic algorithm.

Index Terms—Optimization, Biogeography Based Optimization, Pattern Search.

I. INTRODUCTION

OPTIMIZATION algorithms are widely employed in various fields of engineering for solving minimization problems such as optimal design, model fitting parameters, resolution of non-linear system of equations. Optimization techniques can be decomposed into two mains methods: the deterministic process and the stochastic or metaheuristic method. Whereas the former leads to accurate results to the closest minimum from an initial guess, the latter aims to track the global minimum while avoiding local optima [1]. Since the resolution of optimization problems aims to determine the global minimum with high accuracy, the combination of a stochastic technique with a deterministic method should improve both the convergence rate and the precision of optimization algorithms.

Associating a deterministic process with a stochastic method can be simply realized by starting the deterministic algorithm for every individual of the stochastic population once their objective function improves [2], [3], [4]. The stochastic method continues with a more favorable solution after the deterministic method converges. With this method, the quality of every possible optimum is greatly improved. However, the diversification in the exploration of the definition set can also be compromised by giving high quality optimum for only few individuals. In order to prevent this issue, a specific mutation operator can be added to the metaheuristic method [3]. Combining those two optimization methods can also be performed by replacing the search step of few Pattern Search deterministic algorithms by the evolution of the Particle Swarm Optimization [5]. In this case, it is the deterministic algorithm which receives the benefit of tracking the global minimum from an evolutionary algorithm rather than the accuracy of the deterministic algorithm which is conferred to the stochastic method. Both coupling methods prove to be

very efficient, robust and accurate to find the global optimum. Although combining these methods could increase the number of evaluations of the objective function per iteration, their convergence rate is usually faster than a single evolutionary algorithm.

In this paper, we propose to combine a stochastic optimization technique and a deterministic process with a strategy based on the behavior of preys and predators. Both techniques runs independently in parallel until the deterministic algorithms converge. After their convergence, they are repositioned at the best solutions of the stochastic process whereas the individuals of the metaheuristic method are placed according to a Pareto front between the objective function and their distance to the positions of the deterministic algorithm.

II. COMBINED DETERMINISTIC AND STOCHASTIC OPTIMIZATION TECHNIQUES

The proposed combination between a stochastic and a deterministic optimization process is based on the behavior of predators and preys. At the beginning of the iterative optimization process, the design variable of the deterministic algorithm consists of the best optimum of the metaheuristic technique until the deterministic process possesses a better optimum. Once the deterministic method converges towards one minimum, the design variable of the deterministic process is set to the best optimum of the stochastic method whereas the design variables of the metaheuristic method are set according to a Pareto front between the distance to the position of the deterministic method and the objective function. This proposed strategy re-initializes the deterministic process near a more promising minimum in a similar manner as a predator following its prey. Moreover, the exploration of the stochastic process is focused on favorable regions of the definition set as

a prey trying to evade its predator. The proposed combination method is also detailed in the algorithm 1 where the objective function is denoted f , the set of the design variable \mathbf{x} , and the subscripts d and s stand for deterministic and stochastic respectively.

Data: Initialization of both optimization techniques

Result: Combined deterministic and stochastic algorithm

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for  $n_{ite} = 1$  to  $N_{max}$  do
  Perform the evolution of  $\mathbf{x}_d$  and  $\mathbf{x}_s$  with the chosen
  evolutionary algorithm and the deterministic method;
  while Deterministic method did not converge once do
    if  $f_d$  worse than  $f_s$  then
      |  $\mathbf{x}_d$  is set to the best of  $\mathbf{x}_s$ ;
    end
  end
end
if every deterministic method converge then
  1. Keep the elitist design variables;
  2. Set  $\mathbf{x}_d$  to the best of  $\mathbf{x}_s$ ;
  3. Set  $\mathbf{x}_s$  according to a Pareto Front between the
  objective function and the distance to  $\mathbf{x}_d$ ;
end
end
end

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Algorithm 1: Combination of a metaheuristic method and a deterministic technique. The objective function is denoted f , the set of the design variable \mathbf{x} , and the subscripts d and s stand for deterministic and stochastic method respectively.

III. RESULTS AND DISCUSSION

The proposed combination of a stochastic algorithm and a deterministic technique is applied for coupling the Biogeography Based Optimization (BBO) to the Pattern Search (PS). The BBO algorithm [6] is a metaheuristic method similar to the genetic algorithm with a specific function for the crossing operation. The PS, introduced in [7], is a direct search optimization method. It is a class of deterministic method which does not compute any approximation or exact evaluation of partial derivative of the objective function.

The performances of the combined PS-BBO are evaluated by minimizing different test functions detailed in [8]. The test functions possess 10 design variables or their maximum allowed according to their definition. The BBO is composed of 50 individuals with a mutation probability of 20 %. Three distinct PS are associated to the BBO. Their initial step size is set to 80 % of the definition set that becomes half of the actual step size during the pull step operation. The PS is supposed to reach its convergence when the norm of the step size is below 10^{-6} . Both algorithms are randomly initialized within the definition set. They contain one elitist individual. The maximum number of iterations is set to 5 000.

In figure 1, the combined PS-BBO is developed for minimizing the Griewank function which holds many minima within a wide definition set. After 1362 iterations, the combined PS-BBO reach the global exact minimum of 0 after repositioning 11 times the design variables of the BBO and the

PS according to the proposed strategy based on the behavior of preys and predators. Although the number of evaluation of the objective function doubles, the combined PS-BBO converges faster and with high accuracy toward the global minimum compared with a single BBO.

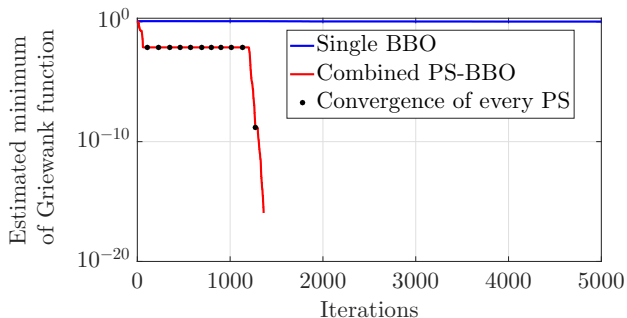


Fig. 1. Evolution of the best minimum determined by the combined PS-BBO and the single BBO in the Griewank. After 1362 iterations, the PS-BBO reach the global exact minimum of 0 which can not be represented in the log-scale.

In table I, the minimum of some test functions with many minima are estimated by the combined PS-BBO which is able to accurately reach the global minimum of every test functions. The proposed combination can significantly improve the performance of optimization algorithms in terms of convergence speed and accuracy. In the final paper, the proposed combined PS-BBO will be employed for electrical machine design.

TABLE I
PERFORMANCES OF THE PROPOSED COMBINED PS-BBO

| Test Function | Minimum (theory) | Optimal design variable (theory) |
|---------------|---------------------------|--|
| Ackley | 6.2×10^{-15} (0) | below 3×10^{-15} (0, ..., 0) |
| Griewank | 0 (0) | below 4×10^{-9} (0, ..., 0) |
| Rastrigin | 0 (0) | below 2×10^{-9} (0, ..., 0) |
| Rosenbrock | 2.4×10^{-4} (0) | $1.00005 < \mathbf{x} < 1.027$ (1, ..., 1) |
| Schwefel 1.2 | 3.2×10^{-69} (0) | below 9×10^{-36} (0, ..., 0) |
| Schwefel 2.21 | 6.8×10^{-13} (0) | below 7×10^{-13} (0, ..., 0) |
| Shekel | -10.1532 (-10.1532) | $4.00004 < \mathbf{x} < 4.00013$ (4, ..., 4) |

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